

REPORT DOCUMENTATION PAGE				Form Approved OMB NO. 0704-0188	
<p>The public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington VA, 22202-4302. Respondents should be aware that notwithstanding any other provision of law, no person shall be subject to any penalty for failing to comply with a collection of information if it does not display a currently valid OMB control number.</p> <p>PLEASE DO NOT RETURN YOUR FORM TO THE ABOVE ADDRESS.</p>					
1. REPORT DATE (DD-MM-YYYY) 27-07-2011		2. REPORT TYPE Conference Proceeding		3. DATES COVERED (From - To) -	
4. TITLE AND SUBTITLE Maximum Likelihood Probabilistic Data Association Multi-Hypothesis Tracker Applied to Multistatic Sonar Data Sets				5a. CONTRACT NUMBER W911NF-10-1-0369	
				5b. GRANT NUMBER	
				5c. PROGRAM ELEMENT NUMBER 611102	
6. AUTHORS P. Willett, Y. Bar-Shalom, S. Schoenecker				5d. PROJECT NUMBER	
				5e. TASK NUMBER	
				5f. WORK UNIT NUMBER	
7. PERFORMING ORGANIZATION NAMES AND ADDRESSES University of Connecticut - Storrs Office for Sponsored Programs University of Connecticut Storrs, CT 06269 -1133				8. PERFORMING ORGANIZATION REPORT NUMBER	
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES) U.S. Army Research Office P.O. Box 12211 Research Triangle Park, NC 27709-2211				10. SPONSOR/MONITOR'S ACRONYM(S) ARO	
				11. SPONSOR/MONITOR'S REPORT NUMBER(S) 57823-CS.11	
12. DISTRIBUTION AVAILABILITY STATEMENT Approved for public release; distribution is unlimited.					
13. SUPPLEMENTARY NOTES The views, opinions and/or findings contained in this report are those of the author(s) and should not be construed as an official Department of the Army position, policy or decision, unless so designated by other documentation.					
14. ABSTRACT The Maximum Likelihood Probabilistic Multi-Hypothesis tracker (ML-PMHT) is an algorithm that works well against low-SNR targets in an active multistatic framework with multiple transmitters and multiple receivers. The ML-PMHT likelihood ratio formulation allows for multiple targets as well as multiple returns from any given target in a single scan, which is realistic in a multi-receiver environment where data from different receivers is combined together. Additionally, the likelihood ratio can be optimized very easily and rapidly with					
15. SUBJECT TERMS ML-PDA, ML-PMHT, Multistatic, Bistatic Active Sonar, Target Tracking, Expectation Maximization, EM					
16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT UU	15. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON Yaakov Bar-Shalom
a. REPORT UU	b. ABSTRACT UU	c. THIS PAGE UU			19b. TELEPHONE NUMBER 860-486-4823

## **Report Title**

Maximum Likelihood Probabilistic Data Association Multi-Hypothesis Tracker Applied to Multistatic Sonar Data Sets

### **ABSTRACT**

The Maximum Likelihood Probabilistic Multi-Hypothesis tracker (ML-PMHT) is an algorithm that works well against low-SNR targets in an active multistatic framework with multiple transmitters and multiple receivers. The ML-PMHT likelihood ratio formulation allows for multiple targets as well as multiple returns from any given target in a single scan, which is realistic in a multi-receiver environment where data from different receivers is combined together. Additionally, the likelihood ratio can be optimized very easily and rapidly with the expectation-maximization (EM) algorithm. Here, we apply ML-PMHT to two multistatic data sets: the TNO Blind 2008 data set and the Metron 2009 data set. Results are compared with previous work that employed the Maximum Likelihood Probabilistic Data Association (ML-PDA) tracker, an algorithm with a different assignment algorithm and as a result a different likelihood ratio formulation.

**Conference Name:** Proc. SPIE Conf. Signal Processing, Sensor Fusion and Target Recognition

**Conference Date:** April 01, 2011



# Maximum Likelihood Probabilistic Multi-Hypothesis Tracker Applied to Multistatic Sonar Data Sets

Steven Schoenecker<sup>a</sup>, Peter Willett<sup>b</sup>, and Yaakov Bar-Shalom<sup>b</sup>

<sup>a</sup>Naval Undersea Warfare Center Division Newport,  
1176 Howell Street, Newport, RI 02841 USA

<sup>b</sup>Department of Electrical and Computer Engineering, University of Connecticut,  
371 Fairfield Way, U-2157, Storrs, Connecticut 06269 USA

## ABSTRACT

The Maximum Likelihood Probabilistic Multi-Hypothesis tracker (ML-PMHT) is an algorithm that works well against low-SNR targets in an active multistatic framework with multiple transmitters and multiple receivers. The ML-PMHT likelihood ratio formulation allows for multiple targets as well as multiple returns from any given target in a single scan, which is realistic in a multi-receiver environment where data from different receivers is combined together. Additionally, the likelihood ratio can be optimized very easily and rapidly with the expectation-maximization (EM) algorithm. Here, we apply ML-PMHT to two multistatic data sets: the TNO Blind 2008 data set and the Metron 2009 data set. Results are compared with previous work that employed the Maximum Likelihood Probabilistic Data Association (ML-PDA) tracker, an algorithm with a different assignment algorithm and as a result a different likelihood ratio formulation.

**Keywords:** ML-PDA, ML-PMHT, Multistatic, Bistatic Active Sonar, Target Tracking, Expectation Maximization, EM

## 1. INTRODUCTION

The Maximum Likelihood Probabilistic Multi-Hypothesis (ML-PMHT) tracker was applied to two data sets provided by the Multistatic Tracking Working Group (MSTWG). The first data set was the TNO Blind data set, a synthetic data set created in 2008 [3] with three moving source-receiver pairs tracking four relatively high strength targets. The second data set was the Metron synthetic data set, featuring five scenarios, each with four transmitters and 25 receivers. This dataset was produced in 2009 and is described in [7].

Tracking results from the ML-PMHT tracker are compared to results on the the same datasets using a different algorithm, the Maximum Likelihood Probabilistic Data Association (ML-PDA) tracker. Results using ML-PDA on these datasets were presented in 2010 [8] and are summarized here.

## 2. ML-PMHT AND ML-PDA TRACKERS

The Maximum Likelihood Probabilistic Multi-Hypothesis algorithm is a batch tracker that assumes some parameterized, deterministic motion for a target. It makes some simple assumptions about a target and its surrounding environment, and then maximizes the likelihood ratio that results from these assumptions. In some ways, it is very similar to the Maximum Likelihood Probabilistic Data Association algorithm (ML-PDA); the main difference between the two algorithms is in the target assignment model. For ML-PMHT, multiple measurements in a single frame (scan) can be assigned to a target; with ML-PDA, at most one measurement can be assigned to a target in a single scan. The ML-PMHT likelihood ratio was first formulated for the PMHT algorithm in [1], [9], [10] and [11]. It was implemented in a maximum-likelihood implementation in a bistatic active application in [13] and [12], which is how we currently employ it.

---

<sup>0</sup>The authors e-mails are steven.schoenecker@navy.mil and {willett, ybs}@engr.uconn.edu.

**Proc. SPIE Conf. Signal Processing, Sensor Fusion, and Target Recognition**, #8050-9, Orlando, FL, April 2011.

Peter Willett was supported by the Office of Naval Research under contract N00014-10-10412.

Yaakov Bar-Shalom was supported by the Army Research Office under contract W911NF-06-1-0467 and the Office of Naval Research under contract N00014-10-1-0029.

## 2.1 ML-PMHT likelihood ratio

The idea behind ML-PMHT is actually very simple. We assume we have a target with some parameterized, deterministic motion in the presence of clutter. With some basic assumptions, we can formulate a likelihood ratio and find the maximum. The ML-PMHT assumptions are:

- A single target is present in each frame with known detection probability  $P_d$ . Detections are independent across frames. (However, it is easy to extend ML-PMHT to multiple targets.)
- Any number of the measurements in the frame can be assigned to the target (again, this is the major difference between ML-PMHT and ML-PDA).
- The kinematics of the target are deterministic. The motion is usually parameterized as a straight line, although any other parameterization is valid.
- False detections are uniformly distributed in the search volume.
- The number of false detections is Poisson distributed with known clutter density.
- Amplitudes of target and false detections are Rayleigh distributed. The parameter of each Rayleigh distribution is known (although the SNR may be tracked [6]).
- Target measurements are corrupted with zero-mean Gaussian noise.
- Measurements at different times, conditioned on the parameterized state, are independent.

Using these assumptions, we can develop the ML-PMHT likelihood ratio as follows. The pdf under the target present hypothesis for the entire set of measurements at time (frame)  $i$  conditioned on the unknown target parameter (vector)  $\mathbf{x}$  is

$$p_1[Z(i)|\mathbf{x}] = \prod_{j=1}^{m_i} \left\{ \frac{\pi_0}{V} p_0^\tau[a_j(i)] + \pi_1 p[\mathbf{z}_j(i)|\mathbf{x}] p_1^\tau[a_j(i)] \right\} \quad (1)$$

Here,  $V$  is the search volume,  $\pi_0$  is the probability that a measurement is from clutter and  $\pi_1$  is the probability that a measurement is from the target. Next,  $p_0^\tau(a)$  is the pdf of a false alarm measurement amplitude conditioned on its exceeding the threshold  $\tau$ ,  $p_1^\tau(a)$  is the pdf of the target measurement amplitude conditioned on its exceeding the threshold, and  $p[\mathbf{z}_j(i)|\mathbf{x}]$  is a target-centered Gaussian. It is also important to note that there are three different notational forms for measurements in this paper. First,  $\mathbf{z}_j(i)$  refers to a single measurement, the  $j^{th}$  measurement in the  $i^{th}$  frame. Next,  $Z(i)$  refers to the set of measurements in the  $i^{th}$  scan. Finally,  $Z$  refers to the set of all measurements in the batch of data (i.e. combining all the scans for  $i = 1, \dots, N_w$ ).

The pdf under the target absent hypothesis is expressed as

$$p_0[Z(i)|\mathbf{x}] = \prod_{j=1}^{m_i} \frac{1}{V} p_0^\tau[a_j(i)] \quad (2)$$

Since one of the assumptions listed above is that measurements at different times, conditioned on the parameterized state, are independent, we can write the likelihood ratio of a batch, or window of data of length  $N_w$  as

$$\frac{p_1(Z|\mathbf{x})}{p_0(Z|\mathbf{x})} = \prod_{i=1}^{N_w} \prod_{j=1}^{m_i} \left\{ \pi_0 + \pi_1 V p[\mathbf{z}_j(i)|\mathbf{x}] \rho_j(i) \right\} \quad (3)$$

where  $\rho_j(i)$  is the ratio of  $p_1^\tau[a_j(i)]$  to  $p_0^\tau[a_j(i)]$ . Thus, the log-likelihood ratio can be written as

$$\Lambda(Z|\mathbf{x}) = \sum_{i=1}^{N_w} \sum_{j=1}^{m_i} \ln \left\{ \pi_0 + \pi_1 V p[\mathbf{z}_j(i)|\mathbf{x}] \rho_j(i) \right\} \quad (4)$$

## 2.2 ML-PMHT Maximization

An advantage of the ML-PMHT log-likelihood ratio formulation is that it can be optimized with a closed-form expression using expectation maximization [4]. As long as there is a linear relationship between the state  $\mathbf{x}$  and the predicted measurement  $\hat{\mathbf{z}}$ , we can write the cost function  $J(\mathbf{x}, Z)$  as

$$J(\mathbf{x}, Z) = \sum_{i=1}^{N_w} \sum_{j=1}^{m_i} \left\{ [\mathbf{z}_j(i) - \mathbf{H}\mathbf{x}]^T \mathbf{R}_{ij}^{-1} [\mathbf{z}_j(i) - \mathbf{H}\mathbf{x}] + \ln(|2\pi\mathbf{R}_{ij}|) \right\} w_j(i) \quad (5)$$

Here,  $\mathbf{H}$  is the measurement matrix,  $\mathbf{R}_{ij}$  is the measurement covariance matrix for the  $j^{th}$  measurement in the  $i^{th}$  scan, and  $w_j(i)$  is the association probability of the measurement. The EM algorithm for this case involves iteratively calculating  $w_j(i)$  and then using this value to solve for the minimum of equation (5) [1]. The expression for  $w_j(i)$  is

$$w_j(i) = \frac{\pi_1 p[\mathbf{z}_j(i)|\mathbf{x}] \rho_j(i)}{\pi_0/V + \pi_1 p[\mathbf{z}_j(i)|\mathbf{x}] \rho_j(i)} \quad (6)$$

As long as the relationship between the state and the predicted measurement  $\hat{\mathbf{z}}$  is linear, as is implied in equation (5), the minimization of this cost function is easily done and is given by

$$\mathbf{x} = \left[ \sum_{i=1}^{N_w} \sum_{j=1}^{m_i} w_j(i) \mathbf{H}^T \mathbf{R}_{ij}^{-1} \mathbf{H} \right]^{-1} \sum_{i=1}^{N_w} \sum_{j=1}^{m_i} w_j(i) \mathbf{H}^T \mathbf{R}_{ij}^{-1} \mathbf{z}_j(i) \quad (7)$$

The state vector  $\mathbf{x}$  in this application is a 4x1 vector,

$$\mathbf{x} = [x_0 \quad \dot{x} \quad y_0 \quad \dot{y}]^T \quad (8)$$

where  $x_0$  and  $y_0$  are the target positions at the beginning of the batch. When the measurements are two-dimensional ( $x$  and  $y$  positions), the relationship between  $\mathbf{x}$  and  $\hat{\mathbf{z}}$  is linear and equation 7 can be used directly. When the measurements are three-dimensional ( $x$  and  $y$  positions and range-rate), this linear relationship no longer holds. At this point, the measurement matrix is no longer a constant, but is a function of  $\mathbf{x}$ . The calculated bistatic range-rate  $\tilde{r}$ , for the state  $\mathbf{x}$ , is given by [2]

$$\tilde{r}(\mathbf{x}) = \frac{1}{2} \frac{(x - x_r)(\dot{x} - \dot{x}_r) + (y - y_r)(\dot{y} - \dot{y}_r)}{R_{TR}} + \frac{1}{2} \frac{(x - x_s)(\dot{x} - \dot{x}_s) + (y - y_s)(\dot{y} - \dot{y}_s)}{R_{TS}} \quad (9)$$

Here,  $(x_r, y_r)$  and  $(x_s, y_s)$  are the positions of the receiver and source,  $(\dot{x}_r, \dot{y}_r)$  are the receiver velocity components,  $(\dot{x}_s, \dot{y}_s)$  are the source velocity components,  $R_{TR}$  is the distance from the receiver to the target, and  $R_{TS}$  is the distance from the source to the target. The predicted measurement in this case is

$$\hat{\mathbf{z}}(\mathbf{x}) = \begin{bmatrix} \mathbf{H}\mathbf{x} \\ \tilde{r}(\mathbf{x}) \end{bmatrix} \quad (10)$$

where

$$\mathbf{H} = \begin{bmatrix} 1 & t & 0 & 0 \\ 0 & 0 & 1 & t \end{bmatrix} \quad (11)$$

In order to get equation 10 into the proper form, it is necessary to linearize the predicted Doppler with a first-order Taylor series expansion.

$$\tilde{r}(\mathbf{x}) \approx \tilde{r}(\mathbf{x}_0) + \nabla \tilde{r}(\mathbf{x}_0)^T (\mathbf{x} - \mathbf{x}_0) \quad (12)$$

Inserting equation 12 into equation 10 produces

$$\hat{\mathbf{z}}(\mathbf{x}) \approx \begin{bmatrix} \mathbf{H} \\ \nabla \tilde{r}(\mathbf{x}_0)^T \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ 0 \\ \tilde{r}(\mathbf{x}_0) - \nabla \tilde{r}(\mathbf{x}_0)^T \mathbf{x}_0 \end{bmatrix} \quad (13)$$

Now, the vector on the right-side of the equation is shifted to the left side of the equation to create a modified measurement, and the relationship between the modified measurement and the state is now is the desired linear form

$$\tilde{\mathbf{z}} = \bar{\mathbf{H}}\mathbf{x} \quad (14)$$

where

$$\tilde{\mathbf{z}} = \hat{\mathbf{z}} - \begin{bmatrix} 0 \\ 0 \\ \tilde{r}(\mathbf{x}_0) - \nabla \tilde{r}(\mathbf{x}_0)^T \mathbf{x}_0 \end{bmatrix} \quad (15)$$

and

$$\bar{\mathbf{H}} = \begin{bmatrix} \mathbf{H} \\ \nabla \tilde{r}(\mathbf{x}_0)^T \end{bmatrix} \quad (16)$$

The gradient is somewhat messy algebraically and is provided here for convenience. For ease of notation, let  $\{x, y\}^{\{R, S\}} = (\{x, y\} - \{x, y\}_{\{r, s\}})$  and  $\{\dot{x}, \dot{y}\}^{\{R, S\}} = (\{\dot{x}, \dot{y}\} - \{\dot{x}, \dot{y}\}_{\{r, s\}})$ . Then,

$$\nabla \tilde{r}(\mathbf{x}) = \frac{1}{2} \begin{bmatrix} \frac{y^R(y^R \dot{x}^R - x^R \dot{y}^R)}{R_{TR}^3} + \frac{y^S(y^S \dot{x}^S - x^S \dot{y}^S)}{R_{TS}^3} \\ \frac{t(y^{R^2} \dot{x}^R - x^R y^R \dot{y}^R) + x^{R^3} + x^R y^{R^2}}{R_{TR}^3} + \frac{t(y^{S^2} \dot{x}^S - x^S y^S \dot{y}^S) + x^{S^3} + x^S y^{S^2}}{R_{TS}^3} \\ \frac{x^R(x^R \dot{y}^R - y^R \dot{x}^R)}{R_{TR}^3} + \frac{x^S(x^S \dot{y}^S - y^S \dot{x}^S)}{R_{TS}^3} \\ \frac{t(x^{R^2} \dot{y}^R - y^R x^R \dot{x}^R) + y^{R^3} + y^R x^{R^2}}{R_{TR}^3} + \frac{t(x^{S^2} \dot{y}^S - y^S x^S \dot{x}^S) + y^{S^3} + y^S x^{S^2}}{R_{TS}^3} \end{bmatrix} \quad (17)$$

### 2.3 Doppler Clutter Modification of ML-PMHT

In [8], an “MTI deweighting scheme” was developed for the ML-PDA tracker. This is used when there is information on the distribution of the Doppler from the clutter measurements. In this work, assuming that the clutter Doppler had a zero-mean Gaussian distribution with variance  $a^2$ , the deweighting term was given as

$$\zeta[d_j(i)] = \frac{e^{d_j(i)^2/2a^2}}{\frac{1}{m_i} \sum_{l=1}^{m_i} e^{d_l(i)^2/2a^2}} \quad (18)$$

Since it is desired to compare the performance of ML-PMHT against that of ML-PDA, this deweighting term is incorporated into the ML-PMHT likelihood ratio in the following manner. The likelihood ratio, from equation 4, is re-written as

$$\Lambda(Z|\mathbf{x}) = \sum_{i=1}^{N_w} \sum_{j=1}^{m_i} \ln \left\{ \pi_0 + \pi_1 V p[\mathbf{z}_j(i)|\mathbf{x}] \rho_j(i) m_i \frac{1}{m_i} \right\} \quad (19)$$

The last term in this equation,  $1/m_i$ , is recognized as the MTI deweighting term when there is no information on the Doppler clutter – i.e. the Doppler clutter has a uniform distribution. This term is simply replaced by  $\zeta[d_j(i)]$  to give

$$\Lambda(Z|\mathbf{x}) = \sum_{i=1}^{N_w} \sum_{j=1}^{m_i} \ln \left\{ \pi_0 + \pi_1 V p[\mathbf{z}_j(i)|\mathbf{x}] \rho_j(i) m_i \zeta[d_j(i)] \right\} \quad (20)$$

This form of the ML-PMHT likelihood ratio was used for performance comparison with the ML-PDA algorithm in this work.

### 2.4 ML-PMHT track initialization

Tracks for ML-PMHT were initiated with a grid-based scheme. Over the 2-dimensional positional search volume, points were evenly spaced in both the x and y directions every 1000 meters. At each  $(x, y)$  grid location, the ML-PMHT log-likelihood ratio was evaluated for a variety of velocity directions and magnitudes. Out of all the log-likelihood values resulting from this grid evaluation, the top 200 (along with their corresponding positional and velocity initialization vectors) were retained. The initialization vector that produced the maximum

likelihood ratio was saved, and other initialization vectors that were essentially duplicates of this first vector were removed. The measurements associated with the first maximum LLR value as well as the duplicates were excised from the data, and the LLR values were recomputed. Again, the vector that produced the largest LLR value was saved, and again, duplicate initialization vectors were removed. This process was repeated until no duplicates remained. Then, all the saved initialization vectors were optimized with EM, and the above process of removing any duplicate initialization vectors was repeated until only unique solutions remained. A simple example (with only 10 initial LLR values) is provided to help illustrate this method. Let the top ten LLR values resulting from the grid initialization be

$$\text{LLR vector} = [10 \ 10.5 \ 9.4 \ 14.2 \ 8.1 \ 8.3 \ 12.5 \ 6.8 \ 6.7 \ 10.6] \quad (21)$$

Next,  $\mathbf{x}_{max}$ , the initialization vector that produced the maximum LLR value (in this case 14.2) was selected and saved as a potential track initialization point. Then, association values for all measurements in the batch were calculated with equation (6), with  $\mathbf{x}$  replaced by  $\mathbf{x}_{max}$ . Now, any measurements that had an association value greater than or equal to 0.1 were excised from the batch of data. The LLR was subsequently recomputed on this reduced data set. In this example, let the LLR vector from excised data come out to

$$\text{Excised LLR vector} = [0.2 \ 0.1 \ 9.4 \ 0.0 \ 8.1 \ 8.3 \ 12.5 \ 6.8 \ 6.7 \ 10.2] \quad (22)$$

The ratio between the two vectors is taken. In the case of the example, the ratio vector is

$$\text{LLR ratio} = [0.02 \ 0.01 \ 1.00 \ 0.00 \ 1.00 \ 1.00 \ 1.00 \ 1.00 \ 1.00 \ 0.96] \quad (23)$$

The initial state vectors that correspond to any LLR ratio value less than 0.5 are discarded as “repeats” of the initial maximum LLR solution vector  $\mathbf{x}_{max}$ . In the case of the example, we would discard the first and second initialization vectors (the  $\mathbf{x}_{max}$  vector would also be removed, but it had been previously saved as a track initialization point). The basic idea here is that we are finding all measurements that are associated with  $\mathbf{x}_{max}$ . If we remove those measurements and then recompute the LLR values, any vector  $\mathbf{x}$  that has its LLR value decrease significantly is most likely just a perturbation of  $\mathbf{x}_{max}$ , and when used as an initializer for an optimization routine, will converge to the same point. Since this will result in an identical/duplicate track, it is necessary to remove this solution. This process is repeated iteratively – using the reduced data set, the  $\mathbf{x}_{max}$  corresponding to the new maximum LLR value was selected (in the example this would be the solution corresponding to the LLR value of 12.5), and the process was repeated. In this manner, the top 200 LLR-producing solutions were reduced to 10 solutions. These 10 solutions were then optimized using the EM algorithm, and again, the process of “solution reduction,” as described above by picking the maximum (now optimized) LLR value, excising measurements associated with the  $\mathbf{x}_{max}$  solution, recomputing the LLR vector and taking the ratio was performed. In this, manner, the number of solutions was reduced to four – these were used at each update as potential track initiation points.

This algorithm worked very well, especially when compared to ML-PDA track initialization routines used in [5] and [8], which used much simpler two-point, one-point or grid initialization schemes. These schemes simply took a large number of trial points, created the LLR at all these points, and then optimized  $N$  (usually around 20) of the initialization vectors that corresponded to the top  $N$  LLR values. While this simpler method was successful in finding tracks, it generally initiated more duplicate tracks than the method described above, and was somewhat slower. This slowness was due to the fact it had to optimize more vectors (which is expensive) in order to account for the duplicate tracks that it would likely find.

## 2.5 Track Maintenance

Track maintenance was done exactly the same as it was for the ML-PDA algorithm described in [8]. Briefly, all tracks were given an integer health value between 0 and 5, with all new tracks being assigned a track health value of 2. At each update, each existing track was projected forward and optimized. If the optimized LLR value of the projected point exceeded the LLR threshold, the track health value was incremented, up to a maximum of 5. Otherwise, the track health value was decreased. When the track health value of a track reached 0, the track was terminated. Additionally, if a track update resulted in a speed that exceeded a maximum speed threshold, the track was dropped immediately.

## 2.6 Multitarget Tracking

The ML-PMHT log-likelihood ratio, as given in equation (4), was developed for a single target. Unlike ML-PDA, this likelihood ratio expression is naturally extensible to multiple targets. Consider an example with two targets, with state vectors given by  $\mathbf{x}_1$  and  $\mathbf{x}_2$ . The log-likelihood ratio in this case for a batch of data is given by

$$\Lambda(Z|\mathbf{x}) = \sum_{i=1}^{N_w} \sum_{j=1}^{m_i} \ln \left( \pi_0 + \pi_1 V \left\{ p[\mathbf{z}_j(i)|\mathbf{x}_1] \rho_{1j}(i) + p[\mathbf{z}_j(i)|\mathbf{x}_2] \rho_{2j}(i) \right\} \right) \quad (24)$$

Here, the assumption is made that the probabilities of a measurement originating from target 1 or target 2 are equal (this need not be the case). In this target assignment model, for each scan, any number of measurements can be assigned to clutter, any number of measurements can be assigned to target 1, and any number of measurements can be assigned to target 2. However, it is important to note that any given measurement is either assigned to clutter, to target 1 or to target 2.

There is a practical problem with using this expression, however, in the tracking application. Consider the case where one target exists, and another target appears. In this case, the LLR expression is given by (24). For track declaration, it is necessary to determine if the “addition” to the LLR by the second track is enough to exceed some threshold and declare the track. In this case, the “extra” LLR due to the second track is given by

$$\Delta\Lambda(Z|\mathbf{x}) = \sum_{i=1}^{N_w} \sum_{j=1}^{m_i} \ln \left\{ 1 + \frac{\pi_1 V p[\mathbf{z}_j(i)|\mathbf{x}_2] \rho_{2j}(i)}{\pi_0 + \pi_1 V p[\mathbf{z}_j(i)|\mathbf{x}_1] \rho_{1j}(i)} \right\} \quad (25)$$

The added LLR that we will use to determine if target 2 should be declared is not only dependent on target 2, but also on target 1. This just gets worse the more targets there are. The “extra” LLR value for the  $n^{th}$  target will be a function of all the previous  $n - 1$  targets. It is desirable in terms of track declaration to have a decision on the  $n^{th}$  target depend on data from that target alone, so for actual implementation, multitarget tracking for ML-PMHT was done in a similar fashion to ML-PDA. When updating an existing track or declaring a new track, measurements from other existing tracks were probabilistically excised from the data prior to evaluating the LLR ratio for the track against a threshold. It was possible, however, to use the EM algorithm to optimize for a track while taking into account already existing tracks. This capability was used for ML-PMHT tracking.

## 3. ML-PMHT VS. ML-PDA ON DATA SETS

The ML-PMHT tracker was applied to two synthetic data sets: the TNO ‘blind’ data set from 2008, and the 2009 Metron data set. These results are compared to previously obtained results for ML-PDA.

### 3.1 TNO Blind Results

The TNO Blind data set was created by Pascal de Theije in 2008 [3]. It consists of a single scenario with three source-receiver pairs and four targets. ML-PDA results (originally done as part of [8]) are shown in Figure 1. ML-PMHT results are shown in Figure 2.

Metrics for both ML-PDA and ML-PMHT trackers are shown in Table 1. Both the metrics and the plots show that overall, the performance of the two algorithms is roughly equivalent.

### 3.2 Metron 2009 Results

The second data set analyzed was a synthetic data set developed by Metron in 2009 [7]. This data set consisted of five different scenarios; the ground truth was provided for the first four scenarios. Each scenario was multi-static with four transmitters and 25 receivers.

This was an extremely challenging data set due to the large number of source-receiver pairs, the low probability of detection of the target for a given source-receiver pair (about 0.1) and the relatively large measurement errors [7]. As an example of the difficulty of this dataset, all of the measurements for a window of data for

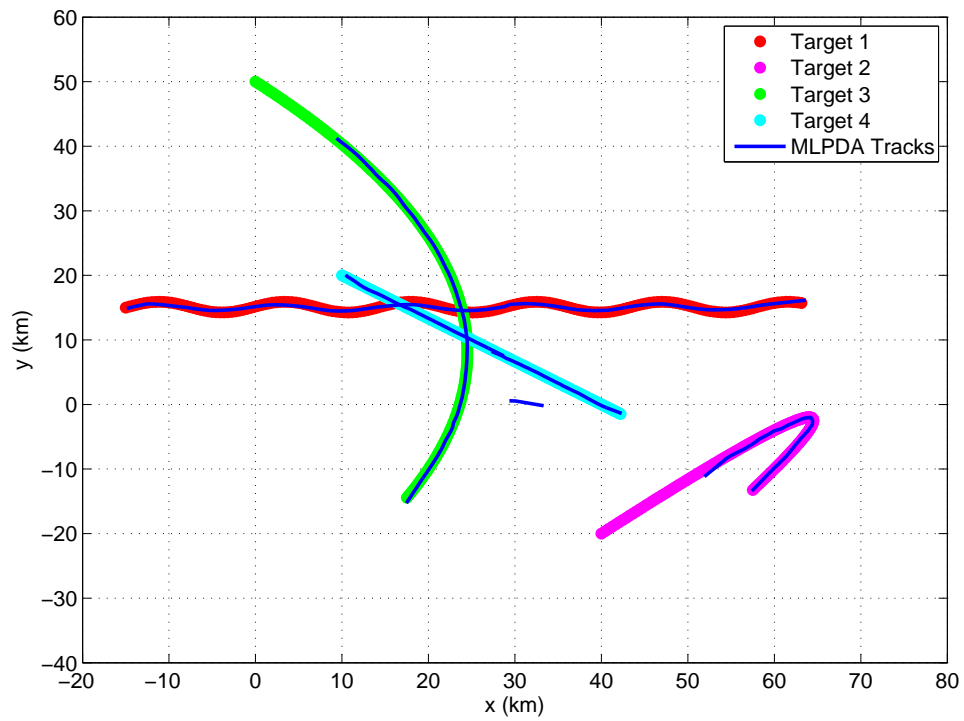


Figure 1. ML-PDA TNO blind data set results

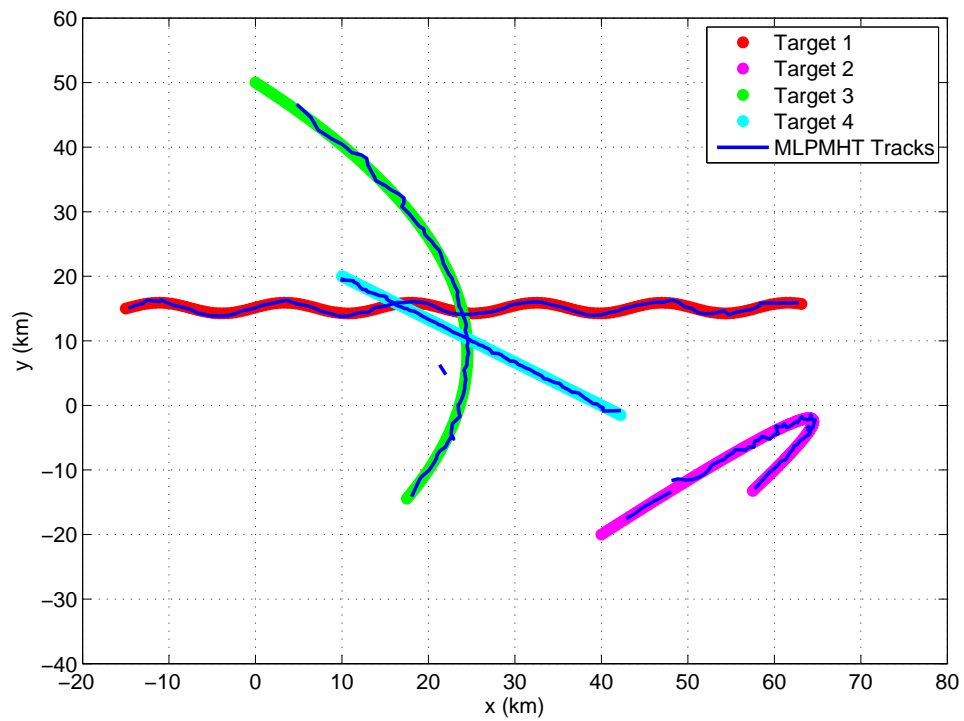


Figure 2. ML-PMHT TNO blind data set results

	Fragmentation	Duplicate Tracks	Percentage of Time in Track	RMSE (m)
ML-PDA Results				
TNO Target 1	0	0	100 %	32.16
TNO Target 2	0	0	80 %	35.65
TNO Target 3	0	0	82 %	20.63
TNO Target 4	1	0	99 %	24.27
ML-PMHT Results				
TNO Target 1	0	0	95 %	37.60
TNO Target 2	3	0	87 %	44.68
TNO Target 3	0	0	86 %	57.29
TNO Target 4	0	0	95 %	19.35

Table 1. ML-PDA and ML-PMHT TNO blind data set metrics

scenario 1 are plotted in figure 3. There are four targets moving in straight lines in this window, yet their trajectories are not visible in the plot.

Metrics for both ML-PDA and ML-PMHT are shown in Table 2. Plots of the five different scenarios for the two algorithms are shown in Figures 4 through 13. As with the TNO data, there do not appear to be any glaringly apparent differences in the performances of ML-PMHT and ML-PDA on this dataset. Quantitatively, in terms of the metrics in Table 2, the results between ML-PDA and ML-PMHT are similar. While there are instances where one algorithm outperforms the other for a certain target in a certain scenario, there is no consistent pattern where ML-PMHT outperforms ML-PDA or vice-versa. Qualitatively, the results seen on the plots are to first order the same. Overall, the performances of ML-PMHT and ML-PDA were good for this dataset. Future work should use Monte Carlo simulations to better evaluate the differences between these two algorithms.

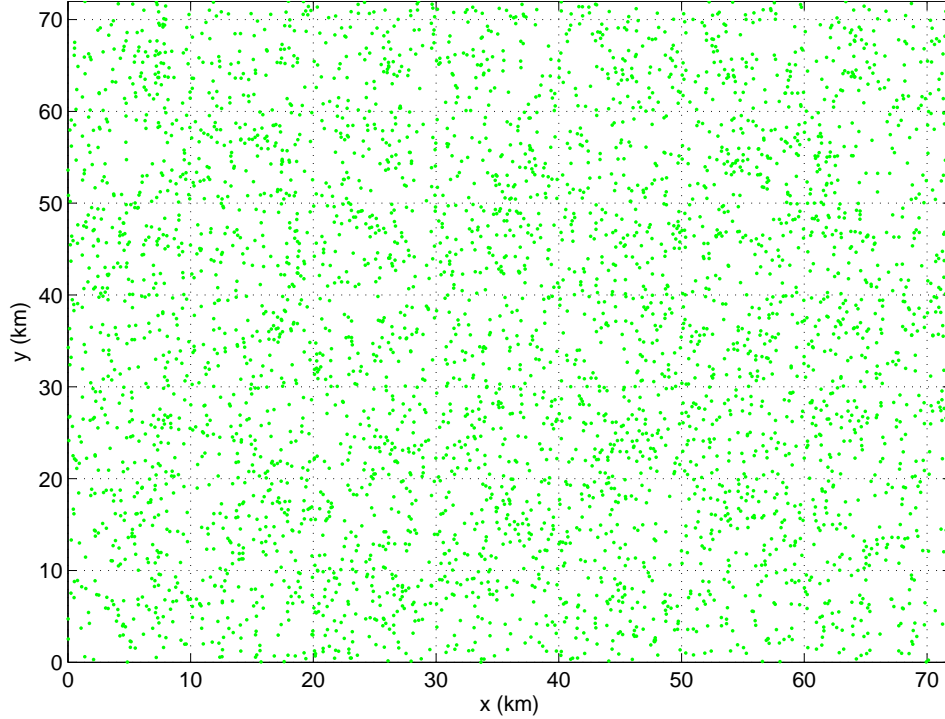


Figure 3. All measurements in a window of data from Metron scenario 1.

	Fragmentation	Duplicate Tracks	Percentage of Time in Track	RMSE (m)
ML-PDA Results				
Metron Scenario 1 Target 1	9	2	33 %	1718
Metron Scenario 1 Target 2	7	4	45 %	1897
Metron Scenario 1 Target 3	11	1	73 %	877
Metron Scenario 1 Target 4	12	0	79 %	937
Metron Scenario 2 Target 1	4	5	67 %	292
Metron Scenario 2 Target 2	7	0	25 %	311
Metron Scenario 2 Target 3	9	0	73 %	435
Metron Scenario 2 Target 4	9	0	73 %	526
Metron Scenario 3 Target 1	10	0	75 %	599
Metron Scenario 3 Target 2	5	0	61 %	159
Metron Scenario 4 Target 1	6	0	69 %	131
Metron Scenario 4 Target 2	8	0	73 %	168
Metron Scenario 4 Target 3	5	1	52 %	214
Metron Scenario 4 Target 4	10	0	75 %	590
ML-PMHT Results				
Metron Scenario 1 Target 1	7	0	39 %	1303
Metron Scenario 1 Target 2	9	0	51 %	1174
Metron Scenario 1 Target 3	14	0	74 %	807
Metron Scenario 1 Target 4	10	1	73 %	730
Metron Scenario 2 Target 1	10	0	60 %	483
Metron Scenario 2 Target 2	5	0	66 %	139
Metron Scenario 2 Target 3	7	0	54 %	316
Metron Scenario 2 Target 4	7	0	64 %	464
Metron Scenario 3 Target 1	6	0	81 %	599
Metron Scenario 3 Target 2	0	0	65 %	43
Metron Scenario 4 Target 1	3	0	68 %	79
Metron Scenario 4 Target 2	2	0	70 %	133
Metron Scenario 4 Target 3	2	0	55 %	65
Metron Scenario 4 Target 4	9	0	58 %	709

Table 2. ML-PDA and ML-PMHT Metron data set metrics

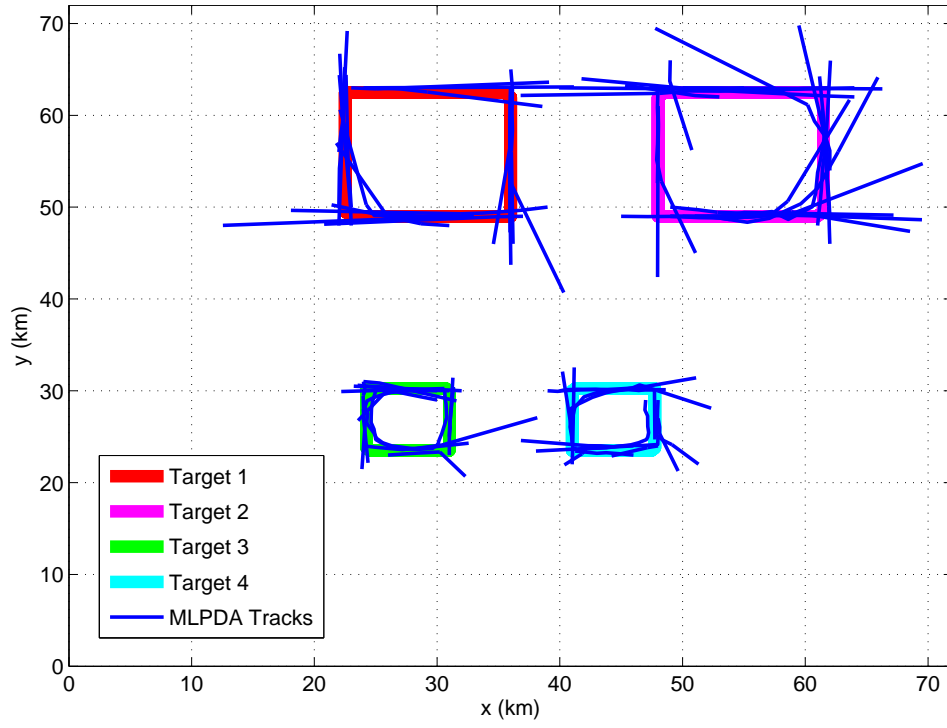


Figure 4. ML-PDA results for Metron scenario 1 (with truth)

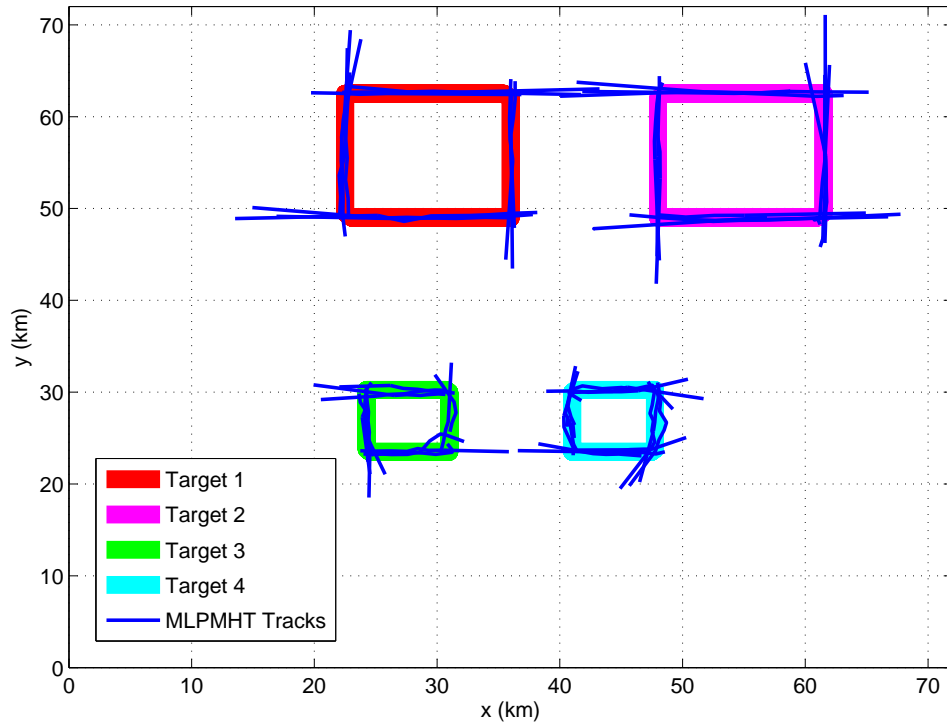


Figure 5. ML-PMHT results for Metron scenario 1 (with truth)

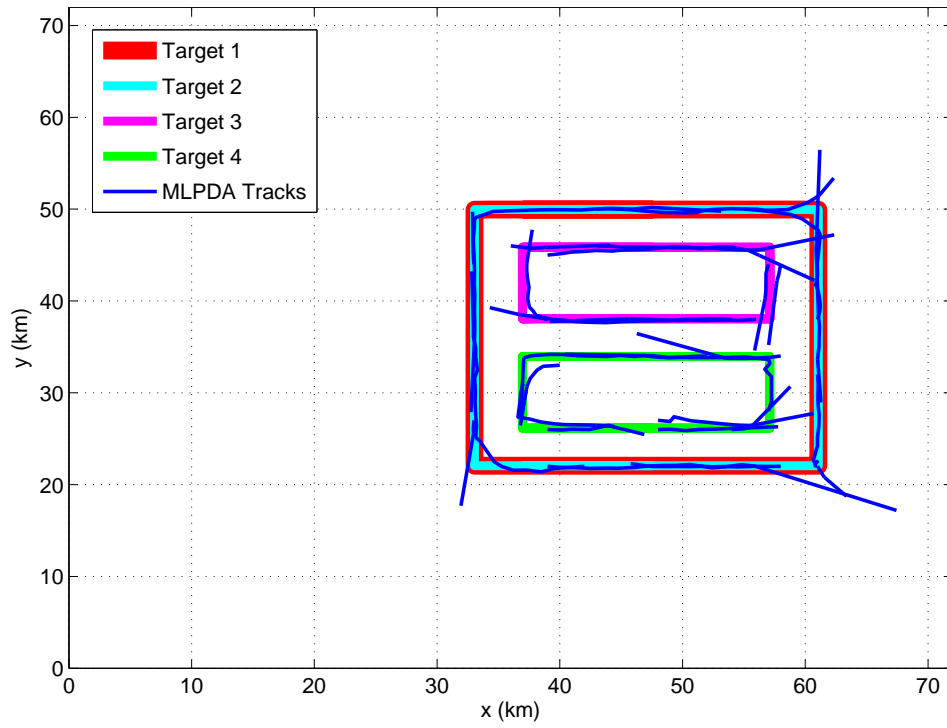


Figure 6. ML-PDA results for Metron scenario 2 (with truth)

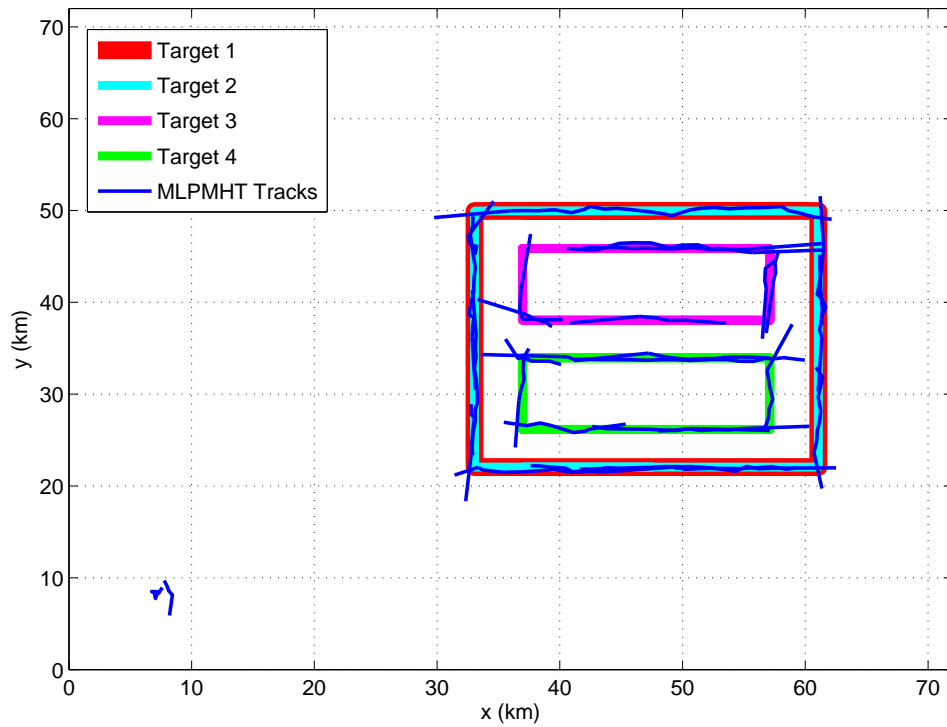


Figure 7. ML-PMHT results for Metron scenario 2 (with truth)

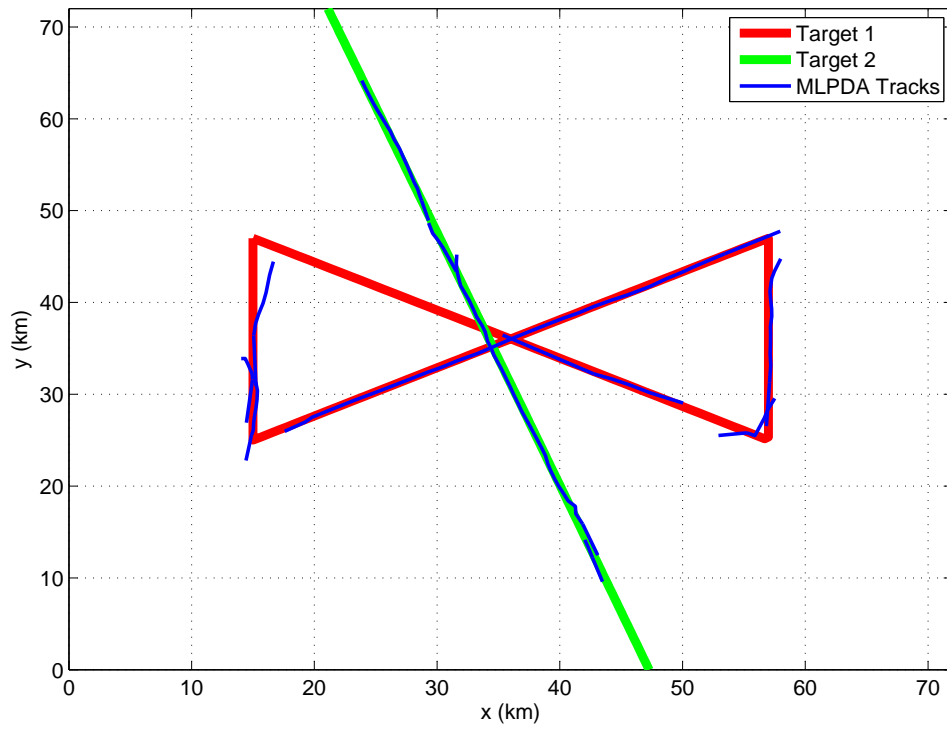


Figure 8. ML-PDA results for Metron scenario 3 (with truth)

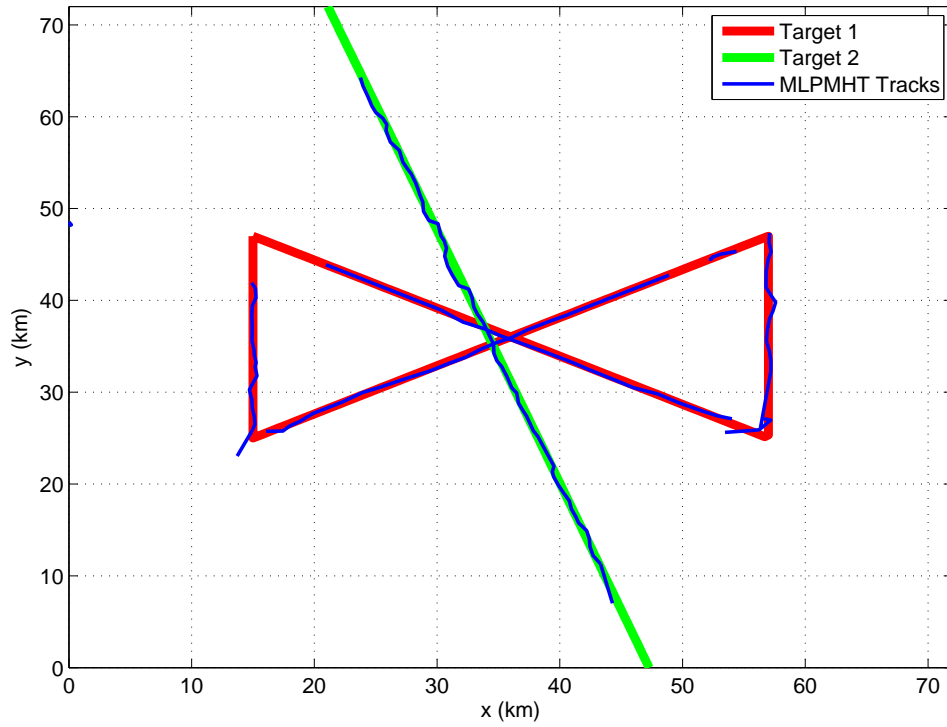


Figure 9. ML-PMHT results for Metron scenario 3 (with truth)

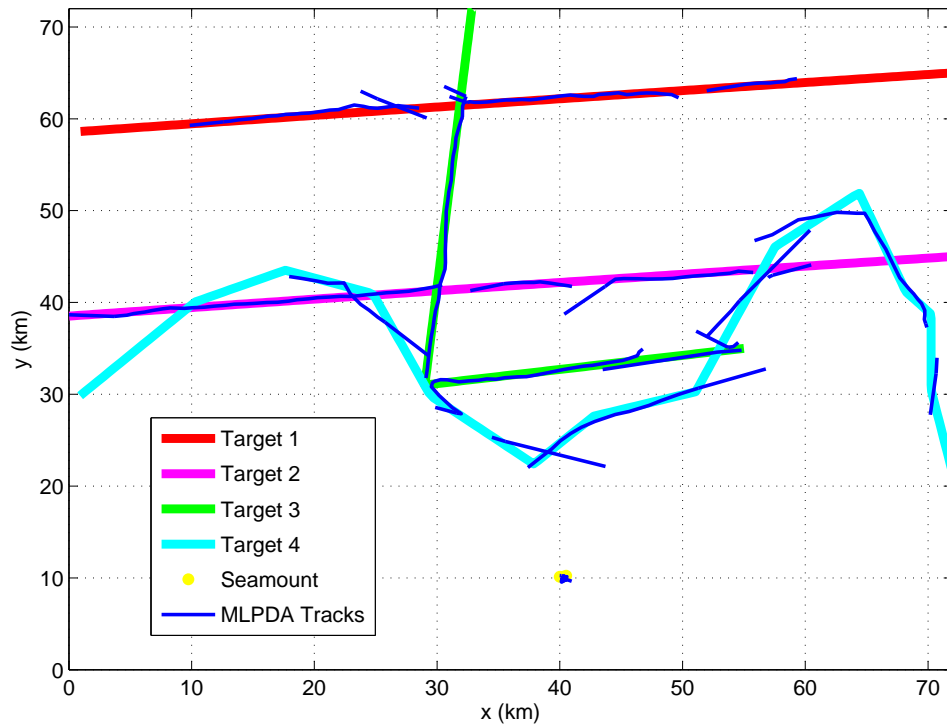


Figure 10. ML-PDA results for Metron scenario 4 (with truth)

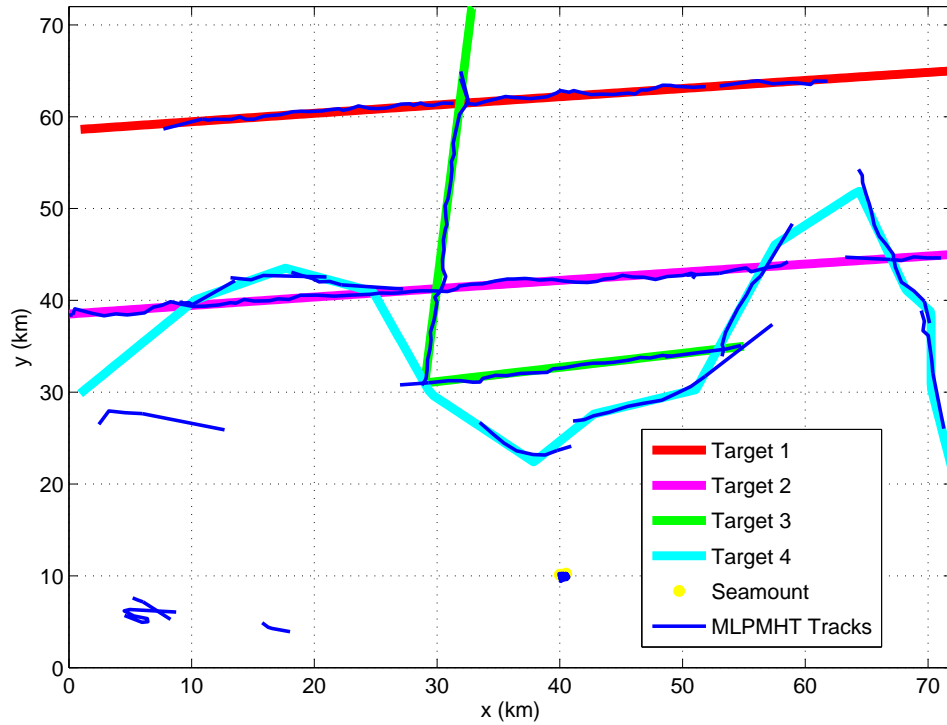


Figure 11. ML-PMHT results for Metron Scenario 4 (with truth)

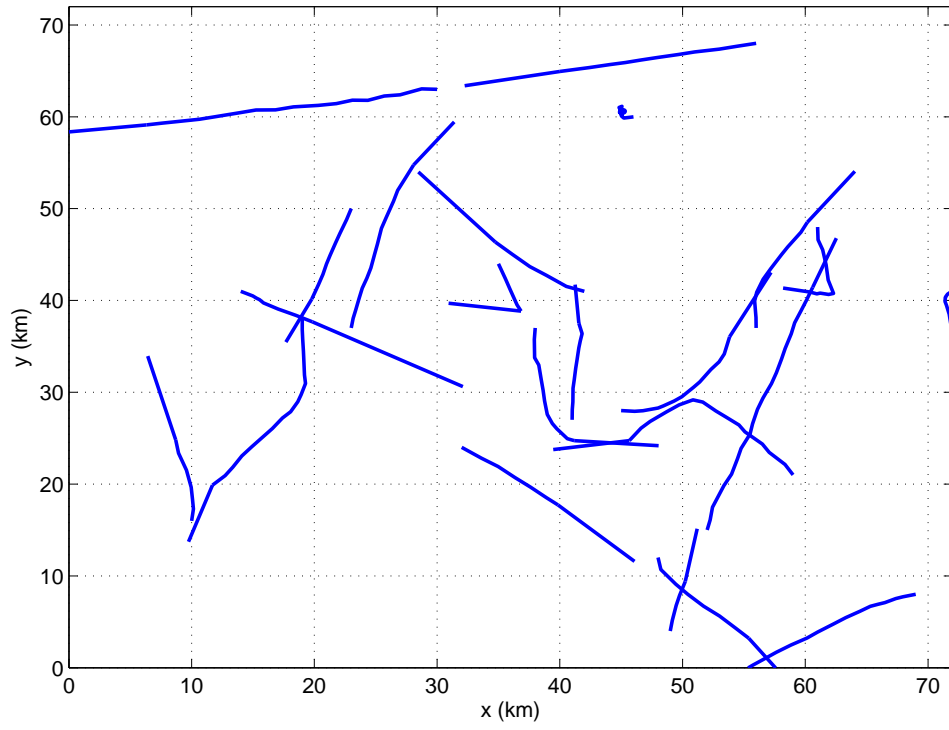


Figure 12. ML-PDA results for Metron scenario 5

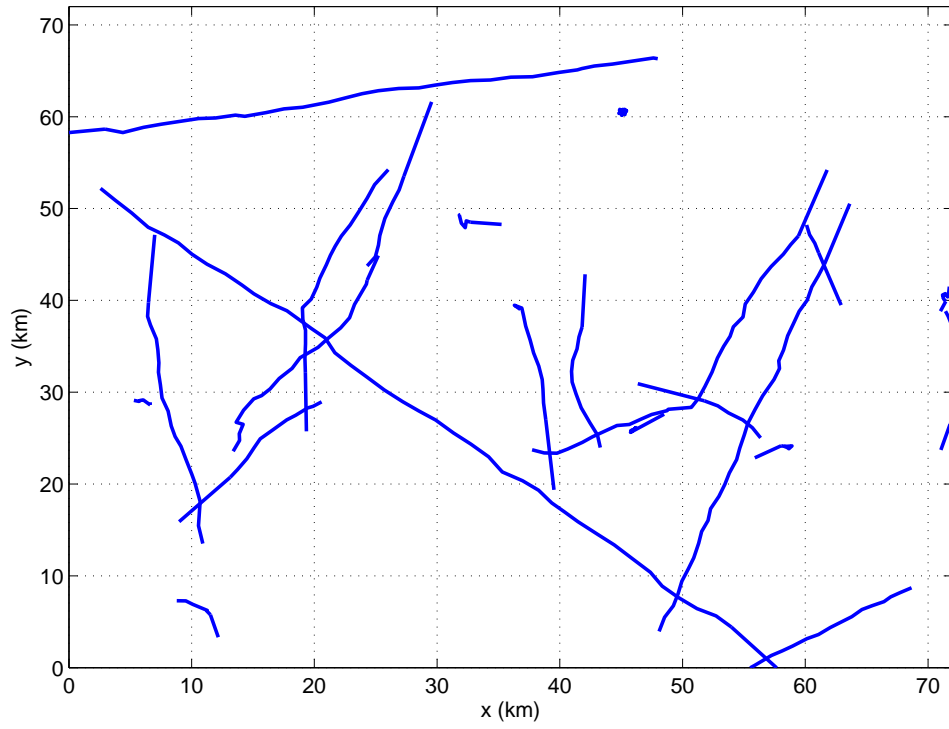


Figure 13. ML-PMHT results for Metron scenario 5

#### 4. CONCLUSIONS

Overall, the ML-PMHT algorithm seems to perform as well as the ML-PDA algorithm for the two datasets considered. The computed metrics are roughly equivalent, and the quality of the tracks, as judged from the plots above, are also roughly equivalent. Future work should be directed towards creating simulation data that can be used for Monte Carlo testing, which will provide a more rigorous comparison between ML-PMHT and ML-PDA.

#### REFERENCES

- [1] D. Avitzour, "A maximum likelihood approach to data association," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 28, no. 2, pp. 560–566, 1996.
- [2] H. Cox, "Fundamentals of bistatic active sonar," BBN Systems and Technologies Corporation, Tech. Rep. W1038, 1988.
- [3] P. de Theije, "Description 'TNOblind' data set for MSTWG," July 2008.
- [4] A. Demster, N. Laird, and D. Rubin, "Maximum likelihood from incomplete data via the EM algorithm," *Journal of the Royal Statistical Society, Series B (Methodological)*, vol. 39, no. 1, pp. 1–38, 1977.
- [5] R. Georgescu, P. Willett, and S. Schoenecker, "GM-CPHD and MLPDA applied to the SEABAR07 and TNO-blind multi-static sonar data," in *Proc of 12th International Conference on Information Fusion*, Seattle, WA, 2009.
- [6] T. Kirubarajan and Y. Bar-Shalom, "Low observable target motion analysis using amplitude information," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 32, no. 4, pp. 1637–1382, 1996.
- [7] K. Orlov, "Description of the Metron simulation data set for MSTWG," 2009.
- [8] S. Schoenecker, P. Willett, and Y. Bar-Shalom, "Maximum likelihood probabilistic data association tracker applied to bistatic sonar data sets," in *Proc. SPIE Conference on Signal and Data Processing of Small Targets*, Orlando, FL, 2010.
- [9] R. Streit and T. Luginbuhl, "A probabilistic multi-hypothesis tracking algorithm without enumeration," in *Proc 6th Joint Data Fusion Symposium*, Laurel, MD, June 1993.
- [10] —, "Maximum likelihood method for probabilistic multi-hypothesis tracking," in *Proc of the Conference on Signal and Data Processing of Small Targets*, Orlando, FL, 1994.
- [11] —, "Probabilistic multi-hypothesis tracking," Naval Undersea Warfare Center, Tech. Rep. TR 10428, 1995.
- [12] P. Willett and S. Coraluppi, "MLPDA and MLPMHT applied to some MSTWG data," in *Proceedings of the 9th International Conference on Information Fusion*, Florence, Italy, July 2006.
- [13] P. Willett, S. Coraluppi, and W. Blanding, "Comparison of soft and hard assignment ML trackers on multi-static data," in *Proceedings of IEEE Aerospace Conference*, Big Sky, March 2006.